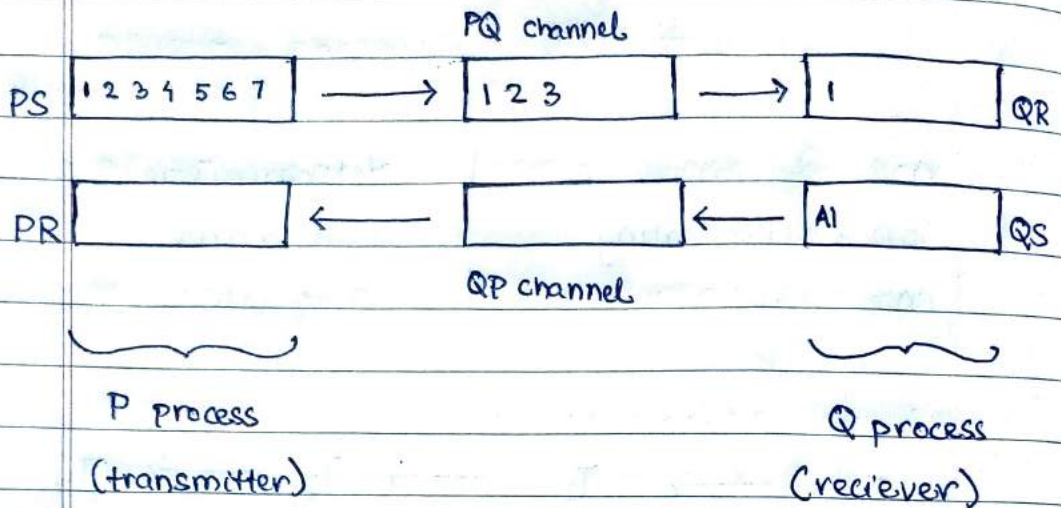


BALANCED SLIDING WINDOW PROTOCOL

its a packet transmission method used for reliable in-order delivery. ex-TCP



in think of P trying to send a file to Q in parts (1-7). Q acknowledges the receipt of a part with an acknowledgement packet (AI).

THE CHANNELS

2 separate channels are PQ and QP.

- they can drop packets
- they can reorder "
- they cant corrupt packets
- even if they could we could use checksum)

THE PROCESSES

2 separate processes P and Q.

- each sends packets through a channel.
- each receives packets through another channel.
- each only sends packet within a "window".
- position of "window" is determined by the packets received by each.

here, P = transmitter process

Q = receiver process

REQUIRED PROPERTIES

① safe delivery

packets received at Q are the ones sent by P.
packets received at P are the ones sent by Q.

② eventual delivery

eventually all packets sent by P are received at Q

eventually all packets sent by Q are received at P.

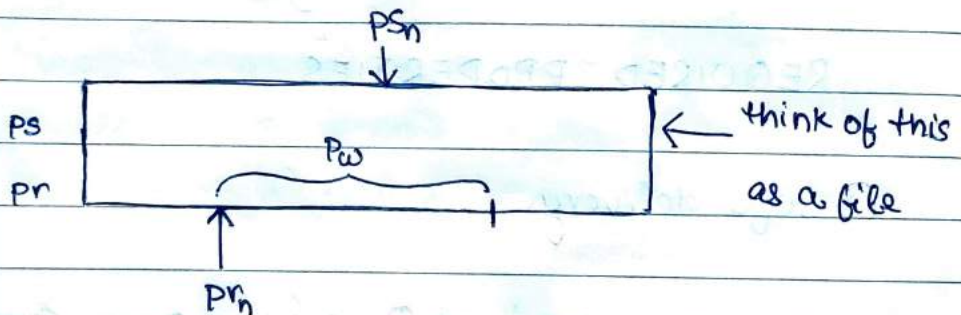
MODELLING AS SYSTEM

consider process P:

state { ps_n - packets to send to Q
 pr_n - packets received from Q (ack.)

functions { ps_n - no. of contiguous packets sent by P
 pr_n - no. of contiguous packets received by P

constant { P_w - window size of P



$$\text{window of P} = [pr_n, pr_n + P_w]$$

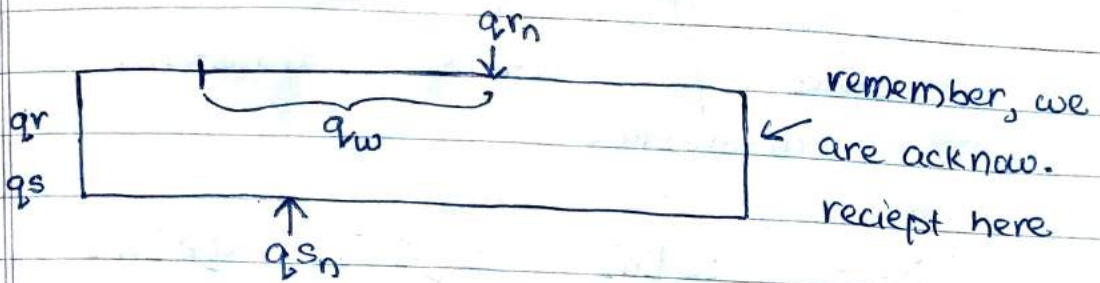
trivial conditions:

$$pr_n \leq ps_n$$

note: P can send any packet in its window, but for determinism, we can add a state variable P_t (turn) indicating which one to send.

similarly for process Q, we have:

$q_s, q_r, q_{sn}, q_{rn}, q_{w} \cdot (q_t)$



window of Q = $[q_{rn} - q_w, q_{rn}]$

trivial conditions:

$$q_{sn} \leq q_{rn}$$

note: in general, ~~with~~

$$\text{window of P} = \begin{cases} [p_{rn} - p_w, p_{rn}] & \text{if } p_{sn} \leq p_{rn} \\ [p_{rn}, p_{rn} + p_w] & \text{otherwise} \end{cases}$$

note: with trivial conditions for P and Q are 4 cases:

- | | | |
|------|--|---|
| X 1. | $p_{sn} \leq p_{rn}, q_{sn} \leq q_{rn}$ | both ^(receivers) acknowledged. ^{not} valid. |
| ✓ 2. | $p_{sn} \leq p_{rn}, q_{sn} \geq q_{rn}$ | p = receiver, q = transmitter. |
| ✓ 3. | $p_{sn} \geq p_{rn}, q_{sn} \leq q_{rn}$ | p = transmitter, q = receiver. |
| ? 4. | $p_{sn} \geq p_{rn}, q_{sn} \geq q_{rn}$ | both transmitters. |

(Can't guarantee eventual delivery for both without acknowledgement).

SYSTEM DEFINITION

$$S = \langle X_s, X_s^0, U_s, \xrightarrow{s} \rangle$$

states initial state actions transitions

$$X_s = \{ ps[n], pq[n], qr[n], pr[n], qp[n], qs[n] \}$$

array of size n (packets)

$$X_s^0 = \{ ps = [1, 2, 3, 4, 5, 6, 7] \}$$

$$pq(i) = \phi \quad \forall i,$$

$$qr(i) = \phi \quad \forall i, \quad \text{acknowledge-ments}$$

$$\rightarrow qs = [-1, -2, -3, -4, -5, -6, -7]$$

or it can
be just ϕ
too

$$qp(i) = \phi \quad \forall i,$$

$$pr(i) = \phi \quad \forall i \}$$

$$U_s = \{ p_{\text{send}}(c_i), p_{\text{recv}}(c_i), \\ q_{\text{send}}(c_i), q_{\text{recv}}(c_i) \}$$

$$\{ p_s, p_q, \dots \} \xrightarrow[\text{s}]{p_{\text{send}}(c_i)} \{ p_{s'}, p_{q'}, \dots \}$$

$$\text{iff } p_{q'} = \{ i \rightarrow p_s(c_i) \} p_q, \text{ and}$$

$$i \in (p_{r_n}, p_{r_n} + p_w], \text{ and}$$

$$i \in [0, n)$$

$$\{ q_s, q_p, \dots \} \xrightarrow[\text{s}]{q_{\text{send}}(c_i)} \{ q_s, q_{p'}, \dots \}$$

$$\text{iff } q_{p'} = \{ i \rightarrow q_s(c_i) \} q_p, \text{ and}$$

$$i \in (q_{r_n} - q_w, q_{r_n}], \text{ and}$$

$$i \in [0, n), \text{ and}$$

$$q_s(c_i) \neq \phi$$

if you set
 $q_s(c_i) = \phi \quad \forall c_i$
 initially

$$\{p_q, q_r, q_s, \dots\} \xrightarrow[s]{q_{recv}(i)} \{p_q, q_r', \dots\}$$

$$\text{iff } q_r' = \{i \rightarrow p_q(i)\} q_r, \text{ and}$$

$$p_q(i) \neq \phi, \text{ and}$$

$$i \in [0, n)$$

$$\{q_p, p_r, \dots\} \xrightarrow[s]{p_{recv}(i)} \{q_p, p_r', \dots\}$$

$$\text{iff } p_r' = \{i \rightarrow q_p(i)\} p_r, \text{ and}$$

$$q_p(i) \neq \phi, \text{ and}$$

$$i \in [0, n)$$

SAFE DELIVERY

$$1. q_r(i) = p_s(i) \text{ or } \phi \quad \forall i$$

$$2. p_r(i) = q_s(i) \text{ or } \phi \quad \forall i$$

} required to prove

we can prove this in 4 steps:

$$1.1 \quad p_q(i) = p_s(i) \text{ or } \phi \quad \forall i$$

$$1.2 \quad q_r(i) = p_q(i) \text{ or } \phi \quad \forall i$$

} \Rightarrow ①.

$$2.1 \quad q_p(i) = q_s(i) \text{ or } \emptyset \quad \forall i \quad \left. \vphantom{q_p(i)} \right\} \Rightarrow \textcircled{2}$$

$$2.2 \quad p_r(i) = q_p(i) \text{ or } \emptyset \quad \forall i$$

1.1 BASE CASE

$$p_q(i) = \emptyset \quad \forall i \quad (\text{trivially true})$$

INDUCTIVE CASE

assume p_q satisfies $\textcircled{1.1}$

the only action U_s that changes p_q is $p_{\text{send}}(i)$.

$$\{p_s, p_q, \dots\} \xrightarrow[s]{p_{\text{send}}(i)} \{p_s, p_q', \dots\}$$

$$\text{iff } p_q' = \{i \rightarrow p_s(i)\} p_q, \text{ and}$$

$$i \in (p_{r_n}, p_{r_n} + p_w], \text{ and}$$

$$i \in ([0, n))$$

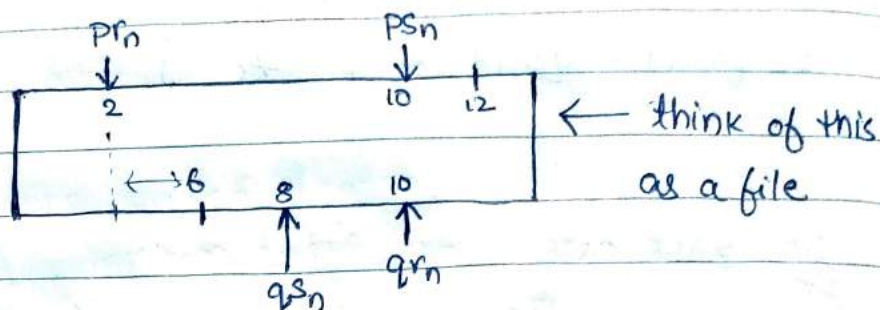
$\Rightarrow p_q'(i) = p_s(i)$ for some i
so $\textcircled{1.1}$ still holds.

Similarly $\textcircled{1.2}$, $\textcircled{2.1}$, $\textcircled{2.2}$ can be proved.

\Rightarrow our system guarantees safe delivery.

(which means no packet is modified in between or reordered).

EVENTUAL DELIVERY



here an example file delivery from P to Q.

P: $pr_n = 2$ $ps_n = 10$ $p_w = 10$ window = $(2, 12]$

Q: $qs_n = 8$ $qr_n = 10$ $q_w = 4$ window = $(6, 10]$

so, P has sent 10 packets

Q has received all 10 packets

Q has sent ACK for 8 packets

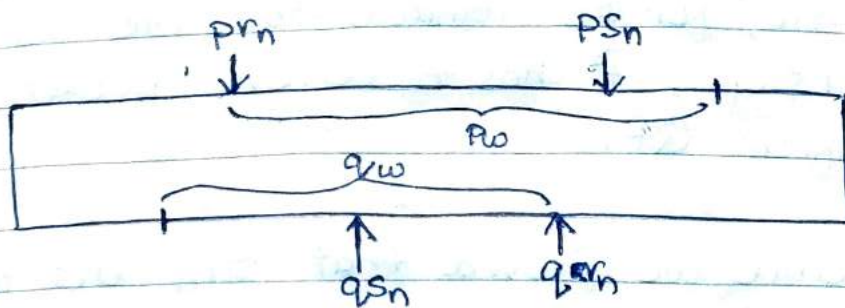
but P has received ACK for only 2 packets

now, what if channel QP drops any packet 2-6, since Q's window is $(6, 10]$ it won't resend those ACK packets.

but, P can't slide its window forward without receiving ACK for packet 2.

so, when $q_w < p_w$

P and Q can stay stuck on channel loss (deadlock).



in order to guarantee eventual delivery despite channel loss, we need to ensure that window of Q atleast includes the start of window of P.

$$P \text{ window} = [p_{rn}, p_{rn} + P_w]$$

$$Q \text{ window} = [q_{rn} - q_w, q_{rn}]$$

$$\text{RTP: } q_{rn} - q_w \leq p_{rn}$$

$$p_{rn} + P_w \geq p_{sn} \quad \text{by window def.}$$

$$p_{sn} \geq q_{rn} \quad \text{by trivial cond.}$$

$$\Rightarrow p_{rn} \geq q_{rn} - P_w$$

$$\Rightarrow p_{rn} \geq q_{rn} - q_w \quad \text{as } q_w \geq P_w$$

$$\Rightarrow q_{rn} - q_w \leq p_{rn} \quad (\text{proved})$$

now, for P's window to slide forward by 1 step, P has to receive packet $q_s(p_{r_n})$ from Q.

since we proved that p_{r_n} lies in window of Q, we can say when Q sends $q_s(p_{r_n})$ P eventually receives it.

similar argument can be made of sliding of Q's window by 1 step.

since we can guarantee sliding of window by 1 step at each step, and that is equivalent to packet reception and its acknowledgement, we can thus say that eventual delivery is guaranteed.